

# Coastal Groundwater Management with Nearshore Resource Interactions

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## Abstract

This paper develops a regional hydrologic-ecologic-economic model of groundwater use and a nearshore ecosystem. Particularly, we model coastal groundwater management and its effects on discharge, nearshore water quality, and marine biota (e.g., indigenous marine algae). We show that incorporating the external effects on nearshore resources increases the optimal steady-state head level. Numerical simulations are illustrated using data from the Kuki'o region on the island of Hawaii. Two different approaches for incorporating the nearshore resource are examined. We find that including algae's market value directly in the objective function calls for lower, albeit slightly lower, water extraction rate in all periods. If a minimum constraint is placed on the stock of the keystone species, greater conservation may be indicated. The constraint also results in non-monotonic paths of water extraction, head level, and water price in the optimal solution.

**Keywords:** groundwater management, submarine groundwater discharge, stock externality, nearshore resources, safe minimum standard, marine algae, dynamic optimization model

**JEL Classification:** Q25, Q28, C61, D62

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## 1. Introduction

In the face of growing water scarcity, groundwater is an important water supply in many areas because of its quality and availability. Many attempts have been devoted to searching for an optimal groundwater management program. Groundwater management is examined extensively by many resource economists including, for example, Renshaw (1963), Burt (1964), Brown and Deacon (1972), and Gisser and Sanchez (1980). This group of pioneer papers addresses the groundwater allocation problem in the context of the theory of the mine, i.e., the aquifer is treated as a bathtub model with a constant rate of recharge. However, this might not be the case for coastal groundwater aquifers.

The problem of groundwater management becomes more complicated when dealing with a coastal aquifer. First, seawater desalination should be added as an additional, albeit high-cost source of freshwater, commonly known as a “backstop” resource. (However, as we shall see, the abundant resource may also be used to facilitate replenishment of an excessively depleted aquifer.)

A second complication regards the natural discharge from an aquifer into the sea. The discharge depends on the amount of groundwater stock; the higher the stock, the higher the discharge, due to the effects of water pressure and ground/surface water interaction. This special characteristic requires a model that allows for endogenous net recharge. The coastal groundwater problem, then, is analogous to the prototypical renewable resource management where natural growth is a function of its own stock. Endogenous net recharge is introduced in groundwater modeling earlier, for example, by Tsur and Graham-Tomasi (1991). The existing endogenous recharge models of groundwater management, however, focus only on the effect of groundwater stock on the change in groundwater net recharge (i.e., the higher

the groundwater stock, the more groundwater discharges into the sea). The external effects of groundwater discharge have not been mentioned in the economic literature.

Groundwater discharge (also called “submarine groundwater discharge”) has many impacts on nearshore environs, e.g., nutrient loads, temperature, or salinity (Johannes 1980, UNESCO 2004). These disturbances may significantly alter the coastal ecological system, endangering unique plants, and animal species with ecological or economic value. In this paper, we are interested particularly in the effects of water quality on Hawaiian indigenous marine algae (also called “limu” in Hawaiian), which is known to be a keystone species of the nearshore ecosystem (Abbott 1978). Some scientific studies examining the effects of water quality on marine algae have been done. For example, Wong and Chang (2000) study the effects of salinity on growth of the commercially important red algae. They find that growth rates for red algae reach their maximum with salinity level 20-25 ppt and decline as salinity increases. Since average seawater salinity is around 30-35 ppt, this implies that an increase in salinity, may induce less growth of the red algae. Although there are many scientific studies in this topic, the links between groundwater discharge and the growth of a specific marine alga have not been examined in any previous literature.

In the next section, we provide a regional hydrologic-ecologic-economic model concerning the interaction between groundwater use and the nearshore ecosystem. Although the model is generalized so that it is applicable to any coastal aquifer system with valuable nearshore resources, particular interest is paid to the Hawaii’s coastal ecosystem.

Incorporating the possibility of using desalinated water either as a backstop resource or as a resource that can facilitate replenishment of an excessively depleted aquifer, the time paths of water extraction for different cases are analytically discussed. We show that, considering the

value of limu, the aquifer's head level must be maintained at a higher level than otherwise in the steady state. Desalinated water will be used only when the shadow price of water in the absence of desalination exceeds the cost of desalination.

Section 3 illustrates the model with a numerical example using the data from Kuki'o area (North Kona Coast, the Island of Hawaii). Two different approaches incorporating limu consideration are discussed. The first approach is to include the market value of limu in the model. This results in the lower groundwater extraction rate, albeit the difference is small as the market value of limu is relatively insignificant compared to the water value. The other approach is to impose a "safe minimum standard" level for limu stock as a constraint. The simulation shows non-monotonic paths of water extraction and head level in the optimal solution for the constraint case. The conclusions are in section 4.

## **2. The Model**

The model structure applied here follows closely to the one in Krulce, Roumasset, and Wilson (1997). Let  $p(x_t)$  represent the price of water, which depends on the total amount of water used ( $x_t$ ). At any time  $t$ ,  $q_t$  amount of water is extracted from the aquifer. The extraction cost increases when the head level is lower because the water must be pumped a farther distance and, in some cases, the water may become brackish and need to be desalinated. The marginal cost of water extraction is assumed to be a positive, decreasing, and convex function in the head level, i.e.,  $c(h) \geq 0$ ,  $c'(h) < 0$ , and  $c''(h) > 0$ .

We assume that an abundant but expensive alternative freshwater source exists. For example, freshwater can be produced by seawater desalination. This is a special feature when dealing with coastal groundwater management, as seawater is abundant in the area. We

denote  $\bar{p}$  as the fixed cost of producing water from desalination, and  $b_t$  is the amount of water produced by desalination technology at time  $t$ .

The change in the head level of aquifer is explained by natural recharge ( $R$ ), amount of water extracted ( $q$ ), and the water discharge to the ocean ( $l$ ). The recharge is assumed constant; or in other words, the average rainfall is fixed over time. As the head level increases, water pressure and surface area from which water can leak increase. Thus, there is more water discharging from aquifer into the ocean when the head level is high. We model discharge as a positive, increasing, convex function of head level, i.e.,  $l(h) \geq 0$ ,

$l'(h) > 0$ , and  $l''(h) > 0$ . The state of the aquifer can be explained as:  $\dot{h}_t = a[R - l(h_t) - q_t]$ , where  $a$  is a conversion factor converting volume to height.<sup>2</sup>

Assume that, at any time  $t$ , limu is harvested at rate  $m_t$ ; and  $p_m(m)$  is the price of limu, which depends on the amount of limu consumed. As the stock level decreases, it is more difficult to find and harvest the limu. The marginal cost of harvesting limu is assumed to be positive, decreasing, and convex with the stock of limu ( $S$ ), i.e.,  $c_m(S) \geq 0$ ,  $c_m'(S) < 0$ , and  $c_m''(S) > 0$ . Natural growth of limu depends on the size of its own stock and water quality,  $G(S, W)$ . With respect to its own stock, the growth is assumed to have the traditional properties; strictly concave and attaining a maximum at a finite value of  $S$ . Examples of the water quality indicator, which may effect the growth of limu, are salinity, nitrogen, and temperature. These indicators are related to the amount of freshwater discharged,  $W = W(l(h))$ . From these relationships, the natural growth of limu can be expressed in terms of limu stock

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<sup>2</sup>  $a$  is specific to the site. It depends on the shape (e.g. length and width), and the porosity of the aquifer.

and head level,  $g(S, h)$ . Assume further that freshwater positively affects limu growth,  $g_h(S, h) > 0$ . The change in limu stock over time is explained by:  $\dot{S} = g(S, h) - m$ .

In the first-best world, a social planner maximizes net social benefit by choosing the paths of groundwater extracted, desalinated water, and limu harvested. However, in the real world, controlling the amount of limu harvested may be difficult administratively and politically. We assume that the government does not have control over limu harvesting. The limu is considered an open access resource, which implies that it will be harvested until its price equals the extraction cost  $p_m(m) = c_m(S)$ . The social planner's problem is to choose the paths of groundwater extraction and desalinated water in order to maximize the social net benefit, which is equal to consumer surpluses, derived from water and limu consumption, minus the costs of obtaining the water and limu. Given discount rate  $r$ , the problem can be written as:

$$\max_{q_t, b_t} \int_0^{\infty} e^{-rt} \left[ \int_0^{q_t + b_t} p(x) dx - c(h_t)q_t - \bar{p}b_t + \int_0^{m_t} p_m(y) dy - c_m(S_t)m_t \right] dt$$

$$s.t. \quad \dot{h}_t = a[R - l(h_t) - q_t]$$

$$\dot{S}_t = g(S_t, h_t) - m_t$$

Suppressing time subscripts, the current-value Hamiltonian is:

$$H = \int_0^{q+b} p(x) dx - c(h)q - \bar{p}b + \int_0^m p_m(y) dy - c_m(S)m + \lambda a[R - l(h) - q] + \theta [g(S, h) - m]$$

... (1)

where  $\lambda \geq 0$  is the co-state variable of the groundwater stock, in terms of the head level, and  $\theta \geq 0$  is the co-state variable of the limu stock. The corresponding first-order conditions are:

$$\frac{\partial H}{\partial q} = p(q+b) - c(h) - \lambda a \leq 0 \quad (=0 \text{ if } q > 0) \quad \text{for all } t \quad \dots (2)$$

$$\frac{\partial H}{\partial b} = p(q+b) - \bar{p} \leq 0 \quad (=0 \text{ if } b > 0) \quad \text{for all } t \quad \dots (3)$$

$$-\frac{\partial H}{\partial h} = c'(h)q + \lambda a l'(h) - \theta g_h(S, h) = \dot{\lambda} - r\lambda \quad \text{for all } t \quad \dots (4)$$

$$-\frac{\partial H}{\partial S} = c_m'(S)m - \theta g_S(S, h) = \dot{\theta} - r\theta \quad \text{for all } t \quad \dots (5)$$

The transversality condition is given by:

$$\lim_{t \rightarrow \infty} \lambda(t)h(t) = 0 \quad \dots (6)$$

Since  $H_{q,q_i} = p'(x) < 0$ , and  $H_{b,b_i} = p'(x) < 0$ ,  $H$  is maximized. Equations (2) to (5) express the optimal conditions of water and limu management. Equation (2) explains that, at the optimum, whenever groundwater is extracted, its marginal user cost (MUC) must be equal to royalty (price minus extraction cost). Equation (3) implies that the desalination technology will not be introduced when the price of water is lower than the cost of desalination. When desalinated water is used, the price of water is equal to the cost of desalination. Equation (4) can be rearranged as:

$$r\lambda + \lambda a l'(h) = \dot{\lambda} - c'(h)q + \theta g_h(S, h) \quad \dots (7)$$

Equation (7) explains that the marginal cost of water conservation (the left-hand side) should be equal to the marginal benefit (the right-hand side). The cost of water conservation consists of the forgone interest from royalty and the monetary value of increased discharge if the water is not extracted. The marginal benefits of water conservation include the change in royalty, the decrease in extraction cost, as the head level is higher, and the benefit of water conservation on limu growth. The equality between the marginal benefit and the marginal

cost of water conservation must hold for all time. Equation (5) has the same interpretation as equation (7), but in terms of limu conservation. The condition requires the equality between the marginal cost of limu conservation and the marginal benefit.<sup>3</sup>

Without desalination, the steady state head level can be obtained by solving the first-order conditions given that  $b=0$ ,  $\dot{p} = 0$ ,  $\dot{h} = 0$  ( $q=R-l(h)$ ), and  $\dot{\theta} = 0$ . The optimal condition at the steady state can be written as:

$$p(q) = c(h) - \frac{a(R-l(h))c'(h)}{r + al'(h)} + \frac{ag_h(S, h)\theta}{r + al'(h)} \quad \dots (8)$$

Assuming that  $g_{hh}(S, h) \leq 0$ , the derivative of the right-hand side of (8) with respect to the head level is unambiguously negative (see appendix A). Since the derivative of the left-hand side is positive, the steady state head level that solves equation (8),  $h^*$ , is unique.

However, if desalination technology is available, it will be introduced when the water price is high enough (i.e., when the shadow price of water in the absence of desalination exceeds the cost of desalination). From equation (3), when desalination technology is used, water price will be constant and equal to  $\bar{p}$ . Substituting  $\bar{p}$  for  $p$  into equation (2) and solving with equations (4) and (5), the steady state condition when desalination is used can be derived as:

$$\bar{p} = c(h) - \frac{a(R-l(h))c'(h)}{r + al'(h)} + \frac{ag_h(S, h)\theta}{r + al'(h)} \quad \dots (9)$$

As the left-hand side of equation (9) is constant and the derivative with respect to  $h$  of the right-hand side is negative (see appendix A), a head level solving equation (9) is unique

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<sup>3</sup> In this case, marginal benefit and marginal cost of limu conservation are not controlled by limu extraction directly. However, the groundwater extraction path should be such that this condition is satisfied.



(called  $h_b^*$ ). Thus, when desalination is being used, the head level will be maintained constant at its optimum,  $h_b^*$ . The optimal groundwater extraction is equal to  $q_b^* = R - l(h_b^*)$ .

**Proposition 1:** The steady state head level is increasing with the value of the limu.

**Proof:** See appendix B.

Proposition 1 indicates that the steady state head level must be maintained at a higher level if demand for limu increases. For example, when there is no externality, the last terms on the right-hand side of equations (8) and (9) disappear ( $\theta = 0$ ). Since the last terms are positive,  $h^*$  and  $h_b^*$  that solve equations (8) and (9) are lower when there is no externality.

The use of desalination technology depends on the demand for water and the extraction cost. If groundwater is available to satisfy demand without drawing the aquifer down to  $h_b^*$  (i.e.,  $h^* > h_b^*$ , or  $p^* < \bar{p}$ ), desalination will never be applied. In that case, the head level will remain at  $h^*$  in the steady state. On the other hand, if  $h^* < h_b^*$ , the desalination will be used at the steady state. The steady state head level will be at  $h_b^*$ .

The optimal extraction and price paths can be derived by solving equations (2), (4), and (5). The condition for the optimal trajectories can be expressed as:

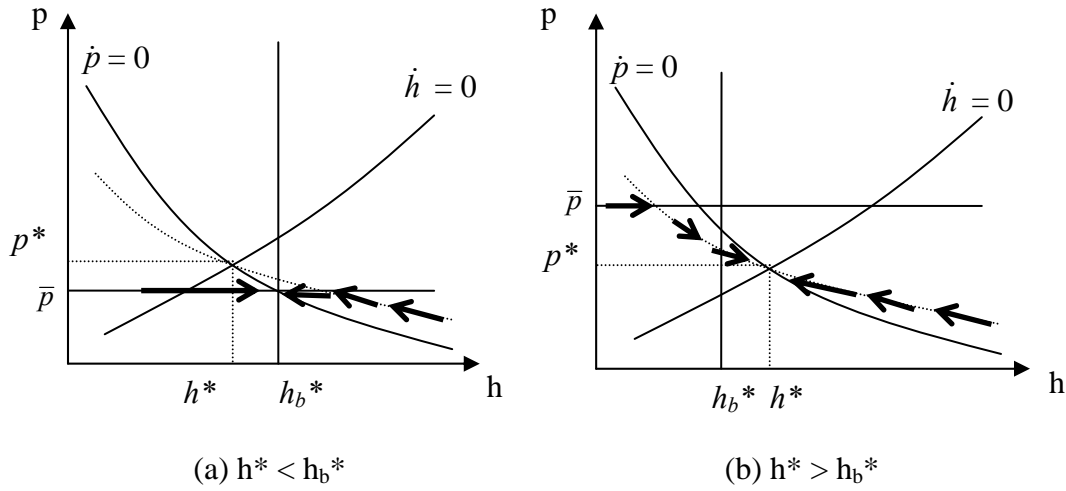
$$p = c(h) + \frac{\dot{p} - a(R - l(h))c'(h)}{r + al'(h)} + \frac{ag_h(S, h)\theta}{r + al'(h)} \quad \dots (10)$$

The optimal condition in equation (11) requires the equality between the marginal benefit of water extraction and the marginal cost, which consists of the extraction cost and the MUC.

The MUC is composed of the forgone benefit from the higher future price, the higher extraction cost in the future, and the external cost of using water on the limu.

From the equation of motion of the head level and equation (11), we can draw phase diagrams explaining the dynamic paths of the head level and water price in different cases

(figure 1). The  $\dot{h} = 0$  locus implies  $q = R - l(h)$ . As the discharge function is increasing and convex with the head level, and the price is decreasing with water used, the  $\dot{h} = 0$  locus is increasing with the head level. For the  $\dot{p} = 0$  locus, if the desalinated water is used in the steady state, the  $\dot{p} = 0$  locus is a vertical line at  $h_b^*$ . If desalination is not used in the steady state, the  $\dot{p} = 0$  locus will have a negative slope (according to equation (8)).



**Figure 1. Phase-diagram showing the dynamic paths of head level and water price.**

From figure 1, given initial head level ( $h_0$ ), there will be an optimal amount of extraction that will yield a monotonic optimal time path of head level and water price (and thus water extraction) for both cases. If the initial head level is higher than the steady state level and  $h^* < h_b^*$  (figure 1 (a)), the price rises (extraction decreases) until it reaches the desalination cost, then desalinated water will be produced. However, if  $h^* > h_b^*$  (figure 1 (b)), the price of water rises (water extraction decreases) and reaches the steady state; desalination will not be used. The problem is more complicated if the initial head level is lower than steady state. In that case, if  $p^* > \bar{p}$  and  $h^* < h_b^*$  (figure 1 (a)), desalination will be used from the beginning until the head level reaches the steady state ( $h_b^*$ ). If, however,  $h^* > h_b^*$  and  $p^* < \bar{p}$

$\bar{p}$  (figure 1 (a)), then the desalinated water will be used in the beginning. Desalination will be stopped when the price of water is lower than desalination cost. In this case, desalination will be used only in the beginning period (so called “frontstop”).

### 3. Application

In this section, we provide a numerical example of groundwater management model considering its nearshore externalities. We choose the Kuki’o area, located along the North Kona coast of the island of Hawaii as a study site. Unless stated otherwise, the data used in this example is based on the study by Duarte (1995)

#### 3.1. Data

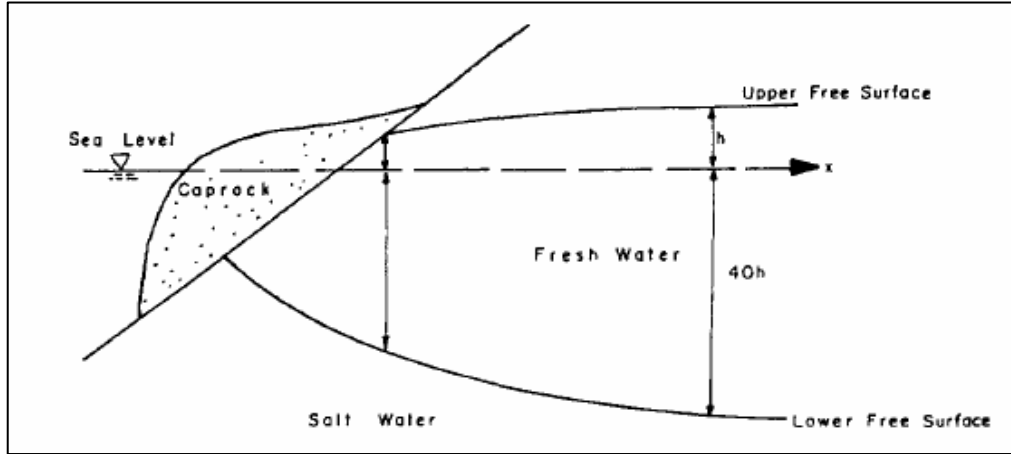
##### Aquifer state equation

Like other coastal aquifers in Hawaii, Kuki’o aquifer exhibits a basal or Ghyben-Herzberg lens (Duarte 1995). It can be thought of as less-dense freshwater floating on an underlying heavier saltwater (see figure 2). The amount of water stored in the aquifer is a function of head, aquifer boundary, as well as aquifer porosity. Duarte (1995) solves the relationship between the volume of basal lens and the head level using a sharp-interface Ghyben-Herzberg relation. The fluid mass balance equation can be expressed by:

$$\dot{h}_t = \frac{2000}{41\theta WL} \dot{V} = \frac{2000}{41\theta WL} (R - l_t - q_t) \quad \dots (20)$$

where  $\theta$  is the porosity;  $W$  is the aquifer width; and  $L$  is the aquifer length.  $\frac{2000}{41\theta WL}$  is a conversion factor converting the volume (thousand cubic meter per year,  $\text{tm}^3$ ) to height (m). For the Kuki’o aquifer, the porosity is 0.3; the aquifer unit width is 6000 m.; and the aquifer length is 6850 m. The state equation for the Kuki’o aquifer can be written as

$\dot{h}_t = 0.00000396(R - l_t - q_t)$ . For notational convenience, the water (e.g., recharge, discharge, or extraction) is presented in units of “thousand cubic meter per year” (tm<sup>3</sup>/y).



**Figure 2. Ghyben-Herzberg lens (Mink 1980)**

### Recharge and discharge

In this paper, we assume away the fluctuation in the net recharge. The average net recharge for the Kuki’o aquifer is equal to 15114 tm<sup>3</sup>/y. Based on Darcy’s law, Mink (1980) derives the structural expression for discharge as a function of the head level. He shows that the relationship can be expressed by:

$$l(h) = kh^2 \quad \dots (21)$$

where  $k$  is a coefficient specific to an aquifer. Due to a lack of data on current discharge, we assume that the aquifer has been drawn down (head level decreases) with the current extraction rate, while it will be built up (head level increases) if no water is extracted. Under these assumptions, the value of  $k$  ranges from 4584.34 to 4935.18. For the purpose of illustration, the discharge function is assumed to be:  $l(h) = 4800h^2$

### Cost of water

The costs of extracting groundwater are primarily due to the cost of energy needed to lift water to the ground level. The unit pumping cost (\$/m<sup>3</sup>) can be expressed as:

$$c = (h_{grd} - h) EC$$

where  $h_{grd}$  is the ground elevation (m),  $EC$  is the energy cost of lifting one unit of water per meter (\$/m<sup>3</sup>/m). For the Kuki'o case,  $h_{grd} = 403.2$  m. With the energy cost at \$0.21/ KWH for the North Kona area,  $EC=0.00083$ . Thus, the unit cost of extraction can be expressed as  $c = 0.00083(403.2 - h)$ . At the current head level ( $h=1.75$ ), water extraction cost is equal to \$0.33/m<sup>3</sup>.

Pitafi (2004), using 2001 data , estimates the unit cost of desalination and transportation of the desalted water to the existing water distribution system at \$7 per thousand gallons. Adjusted for the inflation rate (Department of Business, Economic Development, and Tourism, 2006), we estimate the cost of desalination to be \$7.7 per thousand gallons in 2005 price (\$2/m<sup>3</sup>).

#### Demand for water

This paper models the demand for water with a linear demand function:

$p_t = a - bq_t$ . For the tractability and simplicity, we assume that there is no growth in demand.

The current pumping rate for the Kuki'o aquifer is 1074.4 m<sup>3</sup>/y (540 gallons per minute).

The Public Utility Commission price (retail price) for water in the region is \$1.27 /m<sup>3</sup> (\$ 4.80 / 1000 gallons). However, the water use is expected to increase to 3809 m<sup>3</sup>/y, following the development projects in the area. We calculate the demand function based on the projected water use.<sup>4</sup>

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<sup>4</sup> One may think of this as an average of the long-run growing demand.

Griffin (2006) reviews studies on the elasticity of demand for water. He examines the results in Dalhuisen et al. (2003) and concludes that the long-term residential price elasticity is approximately -0.7 (Griffin 2006, p.309). According to the projected water use, price, and elasticity, the demand for water can be estimated as  $p_t = 3.1 - 0.00048q_t$ . The corresponding consumer surplus is expressed by  $1000(3.1q_t - 0.00024q_t^2)$ .

### Salinity

At the current level of discharge, the average coastal seawater's salinity around the area is approximately 31 ppt (parts per thousand). We assume that if there is no discharge, the salinity level will be at the average level of seawater (36 ppt). Assuming that salinity is linearly related to discharge, the relationship can be expressed as:

$$sal = 36 - 0.00033 l$$

### Limu

In this paper, we choose "limu kohu" (*Asparagopsis taxiformis*) as the species of concern (see figure 3). This is simply because limu kohu is found in the area of Kona coast, its data is available, and it has market value. In 2001, the market value of limu kohu is the highest among all limu (Department of Land and Natural Resources, 2001). In Preskitt (2002), limu kohu is explained as:

*"Plant has creeping basal portion from which soft, fuzzy uprights grow.*

*Found on edges of reef in areas of constant water motion. Only uprights are collected; plants are rinsed thoroughly, soaked overnight, then lightly salted.*

*Upper branches are pounded and rolled into balls the size of a walnut for indefinite storage. Used in small quantities as flavor is penetrating. Added to poke, lomi, and stewed beef. Favorite limu of most Hawaiians."*



**Figure 3. Limu Kohu (Preskitt, 2002)**

According to the Department of Land and Natural Resources (2001), around 1,060 kg of limu kohu were commercially picked in the State of Hawaii (all islands), with a total value of \$19,291. The price of limu kohu is approximately \$18.2/kg. In The same report shows that 15.3% of sea landing (all species) comes from the Island of Hawaii and that out of that amount, 9.5% comes from the area between Keahole to Kawaihae, which is the area of our interest. Applying these ratios to the limu harvest, I infer that 15.4 kg of limu kohu were harvested from this area in 2001.<sup>5</sup>

Due to the lack of data on the stock of limu, we assume that the initial limu stock is 100 kg (one can think of 100 as an index number). The natural growth function of limu is assumed to be in the logistic form, where natural growth depends on the net proportion growth rate ( $\alpha$ ) and the carrying capacity level ( $S_{cc}$ ), i.e.,  $g = \alpha(1 - S / S_{cc})S$ . We assume

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<sup>5</sup> The actual number of limu harvest maybe different due to two factors. First, because limu is not commercially harvested extensively in the area, the actual number may be lower. This is because the proportion data used here represents an all-species harvest, not only limu. Second, including the non-commercial harvest, the actual total harvest must be higher than the commercial harvest alone. With limited data, I assume here that the total harvest is equal to 15.4 kg/year.

that the carrying capacity of limu depends directly on the salinity level<sup>6</sup> and can be explained by  $S_{cc} = 200(31 / sal)$ . For example, at the current level of salinity, the carrying capacity of limu is twice that of the current stock, i.e., 200 kg. The growth rate of limu also depends on the salinity level of the water. We assume further that at the current salinity (31 ppt), 100 kg of limu yields 15.4 kg of limu per year (i.e., at the current harvest rate and salinity level, the stock of limu does not change). Studies by Hoyle (1976) and Wong and Chang (2000) indicate that the growth rates of limu approximately decrease by 50% when salinity increases from 31 ppt to 40 ppt. Assuming that the growth rate is linearly related to the salinity, we can write the function explaining the annual increase in limu as:

$$g = (0.8412 - 0.0172sal)(1 - S / S_{cc})S .$$

Marasco (1974) reviews the elasticity of demand for fish in the U.S. He finds that the elasticity ranges from -0.65 to -0.23. We believe that the elasticity for demand of limu should be more elastic because it is less necessary in terms of the human consumption. However, limu is widely consumed and is a part of culture in Hawaii. This will make the demand more inelastic. For purposes of illustration, we assume that the elasticity of demand for limu is -0.4. The linear demand function for limu kohu in the area is, then, expressed by:  $p_m = 63.63 - 2.95 m$ .

For the cost of harvesting limu, we assume that if the limu is abundant, it requires 0.5 hour to harvest 1 kg of limu. With the open-access assumption ( $S=100$ ,  $c_m=18.2$ ) and the minimum wage rate at \$7.25 per hour, the unit cost can be written as:  $c_m = \left( 0.5 + \frac{201}{S} \right) 7.25$ .

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<sup>6</sup> The carrying capacity concept applied here can represent the competitiveness among different species. When salinity is high, invasive algae can grow better than indigenous algae. Thus, the carrying capacity of the indigenous specie decreases as salinity increases. This effect is separated from the direct effect of salinity on the growth rate.



The parameters and functions used are presented in table 1 and table 2 respectively.

**Table 1. Parameters used in the model**

Parameter	Explanation [units]	Value
$\theta$	Porosity [-]	0.3
$W$	Aquifer width [m]	6000
$L$	Aquifer length [m]	6850
$R_0$	Net Recharge [ $\text{tm}^3/\text{y}$ ]	15114
$L_0$	Current discharge [ $\text{tm}^3/\text{y}$ ]	14700
$h_0$	Initial head level [m]	1.75
$h_{\text{grd}}$	Ground elevation [m]	403.2
$EC$	Energy cost of lifting 1 $\text{m}^3$ of water up for 1 m. [ $\$/\text{m}/\text{m}^3$ ]	0.00083
$\bar{p}$	Desalination cost [ $\$/\text{m}^3$ ]	2
$q_0$	Current water extraction rate [ $\text{tm}^3/\text{y}$ ]	1074.4
$q_p$	Projected water use [ $\text{tm}^3/\text{y}$ ]	3809
$p_0$	Current water price [ $\$/\text{m}^3$ ]	1.27
$\eta$	Long-term elasticity of demand for water [-]	-0.7
$sal_0$	Current salinity level [ppt]	31
$S_0$	Initial limu stock [kg]	100
$S_{cc0}$	Current limu carrying capacity [kg]	200
$m_0$	Current limu harvesting [kg/y]	15.4
$p_{m0}$	Current limu price [ $\$/\text{kg}$ ]	18.2
$\eta_m$	Elasticity of demand for limu [-]	-0.4
$r$	Discount rate [%]	3

**Table 2. Equations used in the model**

Function	Equation
Aquifer state equation	$\dot{h}_t = 0.00000396(R - l_t - q_t)$
Discharge function	$l(h) = 4800h^2$
Unit cost of water extraction	$c = 0.00083(403.2 - h)$
Water demand	$p_t = 3.1 - 0.00048q_t$
Salinity level	$sal = 36 - 0.00033 l$
Natural growth function of limu	$g = (0.8412 - 0.0172sal)(1 - S / S_{cc})S$
Limu demand	$p_m = 63.63 - 2.95 m$
Limu harvesting cost	$c_m = \left(0.5 + \frac{201}{S}\right)7.25$

### 3.2. Scenario Description

Using the above data, we can calculate the optimal water extraction rates, price paths, head levels, and other variables of interest for different scenarios. This paper uses Excel's Solver to solve the dynamic optimization problems. We start by solving for optimal water management without limu consideration (O). Then, the optimal water extraction rates, taking into account the value of limu, are derived (L). However, the value of limu in the calculation represents only the value for consumption, ignoring the ecological and cultural value.

Realizing that the consumption value of limu may not adequately capture its importance, we alternatively model the limu concern by imposing a minimum constraint on the level of limu stock. Particularly, the stock of limu is restricted such that it cannot be depleted by more than a certain percent of the current level. The minimum stock constraint can also be thought of as

a “safe minimum standard” corresponding to the precautionary principle. The precautionary principle is commonly suggested by ecologists and environmental economists as a principle dealing with the problem of species loss, which involves ecological complexity and irreversibility (Harris 2005, p.147). For example, Gollier, Jullien, and Treich (2000) show that the higher scientific uncertainty about future risk is, the higher prevention society should take.<sup>7</sup> In this paper, we run the simulations with three different levels of constraint on minimum stock of limu. Three scenarios include the restrictions that limu stock has to remain at least 50% (L50), 75% (L75), and 90% (L90). Table 3.3 summarizes the scenarios and their explanations.

**Table 3.3. Description of the scenarios**

Scenario	Description
Baseline (B)	Using current extraction rate for all time
Projected (P)	Using projected extraction rate for all time
Optimal (O)	Optimization without limu consideration
Limu (L)	Optimization considering the value of limu
Limu 50 (L50)	Optimization with the constraint that limu stock has to remain at least at 50% of the current level (cannot be depleted by more than 50%)
Limu 75 (L75)	Optimization with the constraint that limu stock has to remain at least at 75% of the current level (cannot be depleted by more than 25%)
Limu 90 (L90)	Optimization with the constraint that limu stock has to remain at least at 90% of the current level (cannot be depleted by more than 10%)

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<sup>7</sup> Mathematically, they show that precautionary principle (as defined by previous prevention effort) is an optimal policy iff the inverse of marginal utility is concave.

### 3.3. Results

For most scenarios, we present the optimization results for 100 years.<sup>8</sup> However, due to the increase in the number of constraints and the limitation of the optimization software, the results of the L90 scenario will be shown for 80 years. If there is no groundwater extraction, the groundwater aquifer will be built up and reach the steady state at 1.77 m in 14 years. The salinity level is 31.01. Limu stock increases to 101.07 kg. With the current groundwater extraction rate, the steady state head level will be at 1.71 m, lower than its current level. It will be reached after 32 years. The salinity level increases from 31 ppm to 31.37 ppm, while the stock of limu decreases from 100 kg to 90.4 kg. However, with the projected extraction rate, the head level will be drawn down to its steady state, 1.54 m, after 50 years. The salinity level increases to 32.27 ppm, and the stock of limu decreases by almost half (to 55.9 kg) in 100 years.

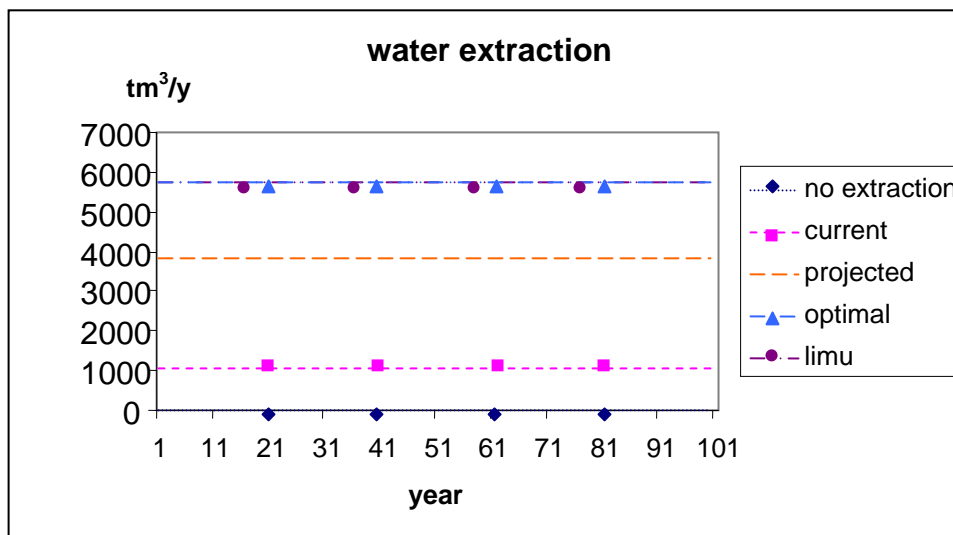
The optimal water management without the limu consideration involves high groundwater extraction rates. The extraction rate starts from 5763.67  $\text{m}^3/\text{y}$  and slightly decreases to 5763.06  $\text{m}^3/\text{y}$  in 100 years. The price increases very slightly from \$0.3334 to \$0.3337 per  $\text{m}^3$ . As the price is always lower than the cost of desalination, desalination will never be exercised. The steady state head level in the optimal scheme is 1.40 m and will be reached in 65 years. The salinity level increases to 32.91 ppm, while the stock of limu decreases considerably to 46 kg in 100 years.

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<sup>8</sup> Because the numerical example is illustrated by using a finite time simulation, I encounter the problem of extensive resource exploitation near the end period. In order to avoid the problem, I run the model with various lengths in order to confirm that each scenario reaches its steady state. I find that the steady state is reached within 100 years in all scenarios. In most cases, the model is run for 200 years. The results from first 100 years are reported.

### 3.3.1. Accounting for the value of the stock externality

Accounting for the value of the limu stock, optimal extraction rates are slightly lower than those of the case without limu. In this case, the extraction rate starts from 5763.64  $\text{tm}^3/\text{y}$  and decreases to 5763.04  $\text{tm}^3/\text{y}$  in 100 years. The steady state head and salinity level, however, are the same as in the previous case. The stock of limu is equal to 46 kg after 100 years. With the lower extraction rate, the price of water is only slightly higher than that of the case without limu. Figures 4 to 6 show the time paths of extraction rate, head level, and limu stock in each case. Figure 7 compares price paths between the cases with and without limu consideration.



**Figure 4. Time paths of the water extraction rate**

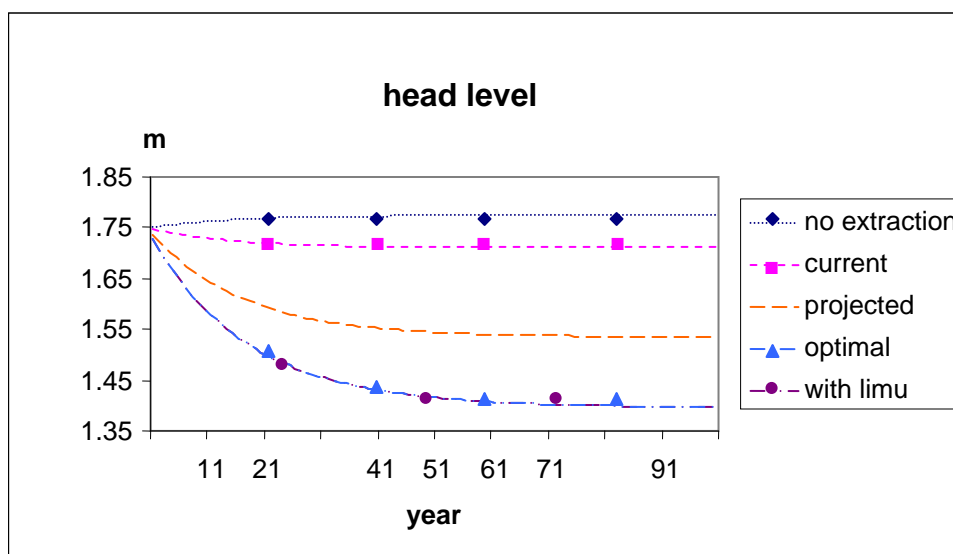


Figure 5. Time paths of the head level

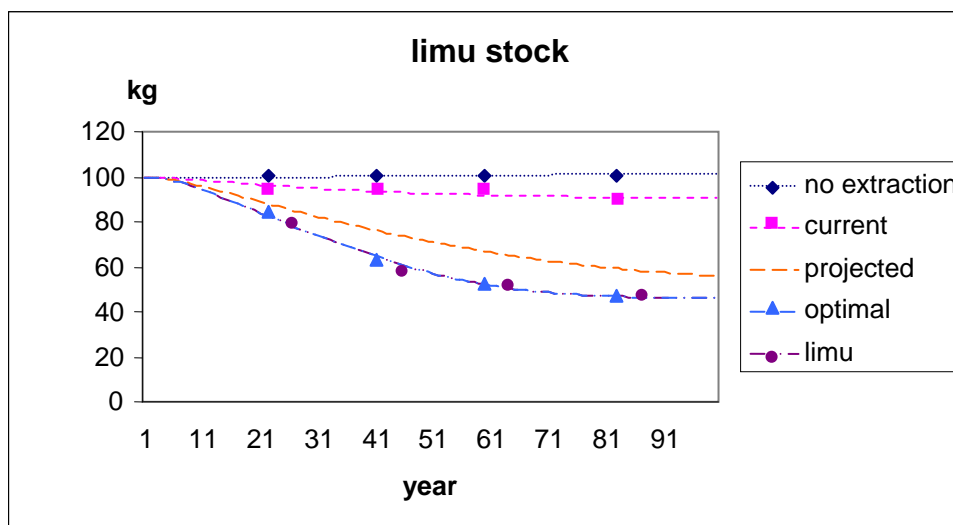
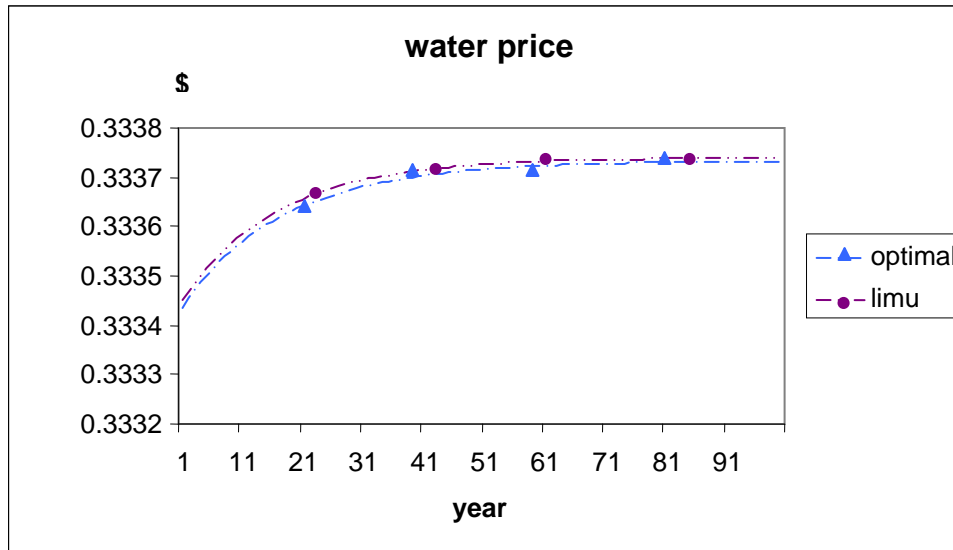


Figure 6. Time paths of the stock of limu



**Figure 7. Time paths of the water price**

From the above results, there are some interesting points that should be mentioned. First, the optimal extraction rate is higher than the projected or current water use. This implies that the current and projected water use is too conservative. It is possible to increase the welfare by extracting more water. This may be partially explained by the fact that the well is currently operated by a private owner. The monopoly type of market for water may explain the lower-than-optimal extraction rate (Perman et al. 2003). Another possible explanation is that the demand for water may be overestimated in the lower ranges of price. In reality, water demand may not be linearly related to price over its entire range. Overestimating the demand for water may result in a high extraction rate. Moreover, this paper models the groundwater aquifer as a bathtub, ignoring the potential effects of extraction rate on the unit extraction cost (e.g., ignoring the impacts of the cone of depression on the extraction cost). This may explain the high extraction rate found in the simulation.

Second, the results show that desalination will never be used in any scenario. As mentioned in the theoretical sections, this happens if the desalination cost is more expensive

than the steady state water price. In this simulation, the water is enough to satisfy the demand with low extraction cost. Since the price of water is always lower than the desalination cost, desalination technology will never be used.

Another interesting result reveals that the extraction rates with the limu consideration are only slightly lower than those in the case where limu is ignored. This is because the value of limu is relatively small compared to the benefit of water consumption. However, it must be emphasized that the value of limu in this example presents only the consumption value and ignores the other values it may have (e.g., ecological or cultural value).

### 3.3.2. With safe-minimum-standard constraints on the limu stock

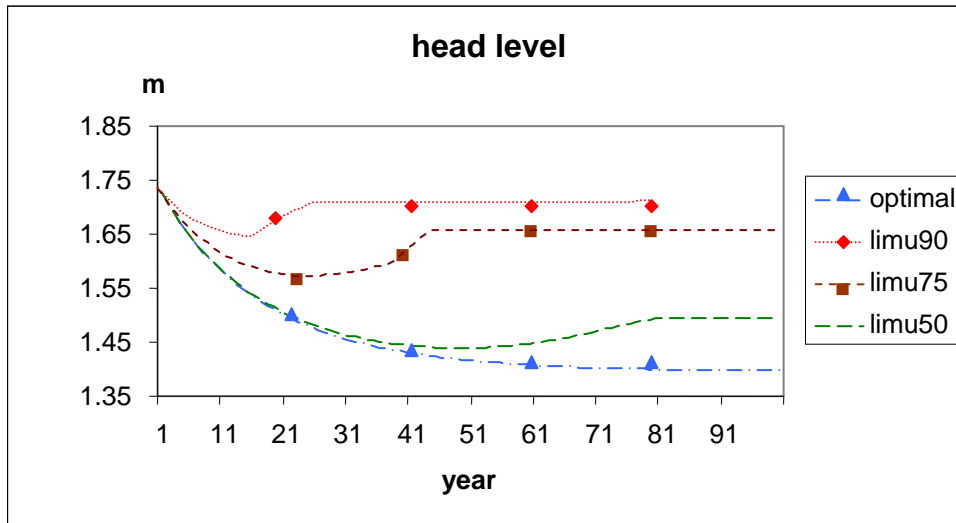
Imposing hard constraints on the level of limu stock leads to more conservative water management. Given that the stock of limu cannot be depleted by more than 50% (L50), the steady state head level is at 1.50 m (compared to 1.40 m from the without constraint case). The steady state extraction rate is around 4380.48  $\text{tm}^3/\text{y}$ . The price of water starts from \$0.35 and reaches \$1.00 per  $\text{m}^3$  in the steady state.

If the limu stock has to remain not less than 75% of the current level (L75), the steady state head level becomes 1.65 m, with the steady state extraction rate around 2000  $\text{tm}^3/\text{y}$ . In this scenario, since the groundwater extraction is low, it is optimal to use desalinated water. Desalination will be first introduced in year 38, when the price of water is equal to desalination cost, \$2/ $\text{m}^3$ . In the steady state, approximately 292  $\text{tm}^3/\text{y}$  of desalinated water will be used.

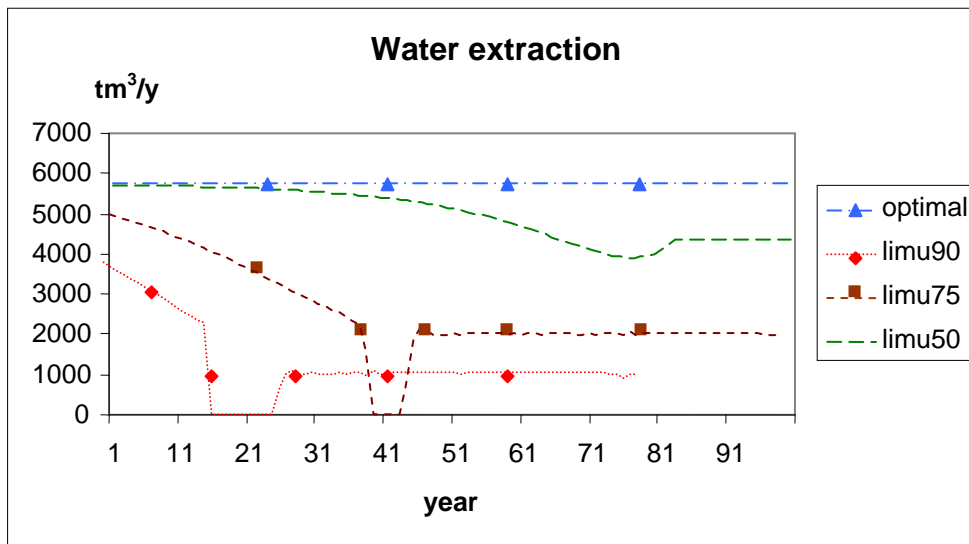
If the limu stock has to be at least 90% of the current level (L90), the steady state head level is 1.71 m. The steady state extraction rate is approximately 1064  $\text{tm}^3/\text{y}$ , while the steady state amount of desalination is around 1227  $\text{tm}^3/\text{y}$  for this scenario (desalination will



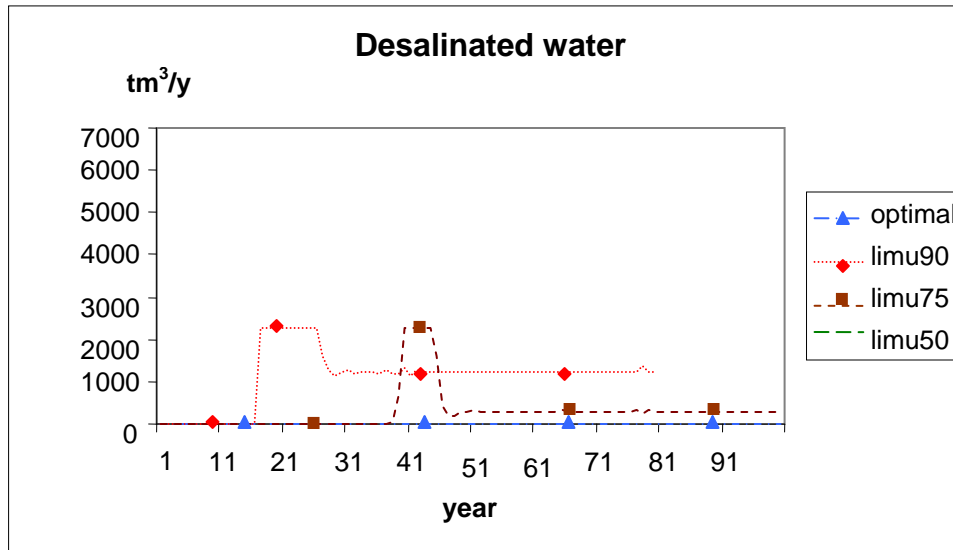
begin after 17 years). Figures 8 to 12 show the time paths of head level, extraction rate, desalination rate, water price, and limu stock for each scenario.



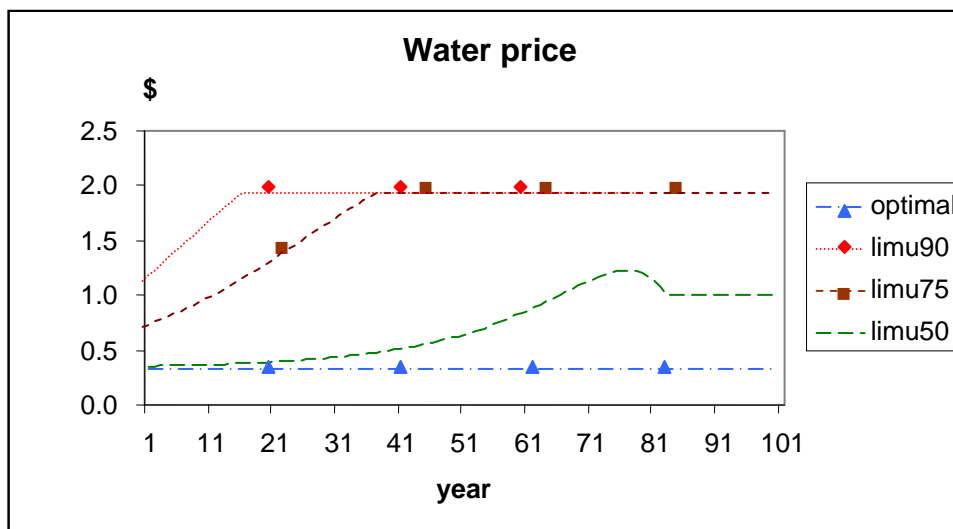
**Figure 8. Time paths of head level with different limu stock constraints**



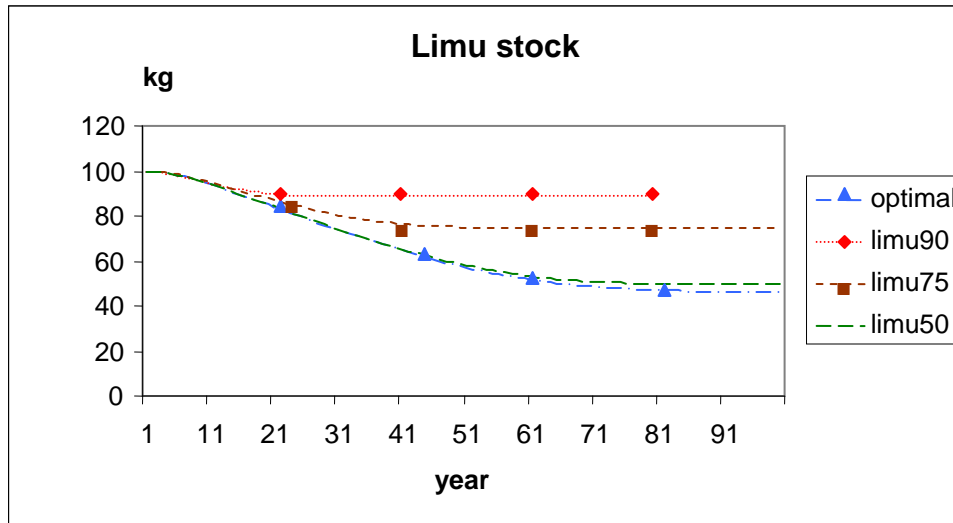
**Figure 9. Time paths of groundwater extraction with different limu stock constraints**



**Figure 10. Time paths of desalination water with different limu stock constraints**



**Figure 11. Time paths of water price with different limu stock**



**Figure 12. Time paths of limu stock with different limu stock constraints**

It is interesting that time paths of the head level, groundwater extraction, desalination, and price can be non-monotonic. With the limu constraint, it is optimal to deplete the aquifer until the head level is lower than the steady state and then conserve the water later on to fill the aquifer until it reaches steady state. Imposing a limu constraint may also induce the use of desalination technology (e.g., L75, L90 cases). In the L75 and L90 cases, it is optimal to stop using groundwater for some periods. The main results (steady states) from all scenarios are summarized in table 4.

**Table 4. Summary of the main results (steady state) from all scenarios**

	<b>B</b>	<b>P</b>	<b>O</b>	<b>L</b>	<b>L90</b>	<b>L75</b>	<b>L50</b>
Head level (m)	1.71	1.54	1.40	1.40	1.71	1.65	1.50
Groundwater extraction (tm <sup>3</sup> /y)	1704.4	3809	5763.06	5763.04	1064	2000	4380.48
Desalination(tm <sup>3</sup> /y)	-	-	-	-	1227	292	-
Salinity (ppm)	31.37	32.27	32.91	32.91	31.36	31.67	32.46
Limu stock (kg)	90.42	55.90	46.01	46.01	90	75	50
Water price (\$/m <sup>3</sup> )	-	1.27	0.33	0.33	2	2	1.00

#### 4. Conclusions

This paper provides a regional hydrologic-ecologic-economic model concerning the interaction between groundwater use and a nearshore ecosystem. We model coastal groundwater management allowing for endogenous net recharge (because of submarine discharge) and an alternative (abundant but expensive) water source (e.g., desalination). The effects of the discharge on the nearshore ecosystem, specifically on the nearshore valuable seaweed, are incorporated. To achieve efficiency, water must be extracted such that its price equals extraction plus user cost. The externality cost is included in the user cost since the externality is determined by the resource stock, not resource extraction. We show four possible patterns of water use. It is also possible that the desalination technology will be introduced in the first period. Considering the effect on limu, the aquifer's steady state head level must be maintained at a higher level than otherwise. Desalinated water will be used

only when the shadow price of water in the absence of desalination exceeds the cost of desalination.

We numerically illustrate the model using the data from Kuki'o area, located on the North Kona Coast of the island of Hawaii. We find that the optimal water extraction rate (without limu consideration) is higher than the current or projected one. This implies that the current/projected extraction is too conservative; and welfare gains can be achieved by extracting more water. This may be partially explained by the fact that the well is currently operated by a private owner. The monopoly type of market causes the lower-than-optimal extraction rate. Another possible explanation is that the demand for water may be overestimated in the lower ranges of price. In reality, water demand may not be linearly related to price over its entire range. Overestimating the demand for water may result in a high extraction rate. Moreover, in this paper, we model the groundwater aquifer as a bathtub, ignoring the potential effects of extraction rate on the unit extraction cost (e.g., ignoring the impacts of the cone of depression on the extraction cost). This may explain the high extraction rate found in the simulation.

In the simulation, two different approaches are applied in order to incorporate the limu consideration. The first approach is to include the market value of limu directly in the objective function. In this case, the water extraction rate is only slightly lower when the value of limu is included. This is because the value of limu is relatively insignificant compared to the high value of water. However, it should be emphasized that the value of limu in this example accounts only for the consumption value and ignores the other possible values, e.g., cultural or ecological value. Desalination technology will never be used because groundwater is enough to satisfy the demand.

The second approach, following the precautionary principle and “safe minimum standard”, models the limu concern by imposing a minimum stock constraint. We find an interesting result that the paths of water extraction, head level, and water price (without desalination) are non-monotonic. It is optimal to deplete the aquifer below the steady state level, following by a conservation period.

Although the model in this paper is developed based on a groundwater-limu framework, it is applicable to other cases that share similar structures, for example, mangrove-nearshore fishery, or forest-wildlife models. In future work, the model can be extended in many ways. For example, in reality, the unit extraction cost may depend not only on the head level, but also on the extraction rate (i.e., via the effects from the cone of depression). Taking into account the presence of the cone of depression will make the model more realistic and thus provide more insight into the actual management problem. One of the most interesting extensions is to model the competition among nearshore species. The growth of limu does not depend solely on the water qualities, but also on the growth of other species, especially fish or invasive limu. Another possible extension is to allow for control on limu harvesting. The comparison between open-access and optimal harvest of limu can be done.

The analysis in this paper is based on a short-run model, e.g., assuming that the wells already exist. The long-run problem of water allocation should involve the location, size and time of well-development. In this paper, we focus only on the first-best solutions. There may be some other constraints that should be taken into account. For example, for administrative purposes or given capacity of water pump limitations, water managers might want to require a constant water extraction rate over some periods in practice. Geographical constraints (e.g., land subsidence) are also an important issue to consider.

## Appendix A.

Given that  $c'(h) < 0, c''(h) > 0, R - l(h) > 0, l'(h) > 0$ , and  $l''(h) > 0$ , the derivative of the left-hand side of equations (7) and (8) with respect to  $h$  is equal to

$$c'(h) + \frac{ac'(h)l'(h)}{r + al'(h)} + \frac{a(R - l(h))c''(h)}{r + al'(h)} + \frac{a^2(R - l(h))c'(h)l''(h)}{(r + al'(h))^2}, \text{ which is unambiguously}$$

negative.

## Appendix B: Comparative static analysis of the steady state head level with respect to $\limu$ value

If desalination is not used, the steady state head level is specified by equation (17), which can be expressed by:

$$0 = c(h^*) - \frac{a(R - l(h^*))c'(h^*)}{r + al'(h^*)} + \frac{ag_h(S, h^*)\theta}{r + al'(h^*)} - p(q) \quad \dots (A)$$

The derivative of  $A$  with respect to  $h$  is equal to

$$A_{h^*} = c'(h^*) + \frac{ac'(h^*)l'(h^*)}{r + al'(h^*)} + \frac{a(R - l(h^*))c''(h^*)}{r + al'(h^*)} + \frac{a^2(R - l(h^*))c'(h^*)l''(h^*)}{(r + al'(h^*))^2} + \frac{ag_{hh}(S, h^*)\theta}{r + al'(h^*)} - \frac{ag_h(S, h^*)\theta(r + al''(h^*))}{(r + al'(h^*))^2} - p'(q)\frac{\partial q}{\partial h^*}$$

Given that  $g_{hh}(S, h) \leq 0$ ,  $A_{h^*}$  is unambiguously negative. The derivative of  $A$  with respect to  $\theta$  is equal to

$$A_\theta = \frac{ag_h(S, h^*)}{r + al'(h^*)} > 0$$

From the implicit function rule, it is easy to see that  $\frac{\partial h^*}{\partial \theta} = -\frac{A_\theta}{A_{h^*}} > 0$ .

If desalination is used in the steady state, the steady state head level is specified by equation (18):

$$0 = c(h^*) - \frac{a(R - l(h^*))c'(h^*)}{r + al'(h^*)} + \frac{ag_h(S, h^*)\theta}{r + al'(h^*)} - \bar{p} \quad \dots (B)$$

The derivative of  $B$  with respect to  $h$  is equal to



$$B_{h^*} = c'(h^*) + \frac{ac'(h^*)l'(h^*)}{r + al'(h^*)} + \frac{a(R - l(h^*))c''(h^*)}{r + al'(h^*)} + \frac{a^2(R - l(h^*))c'(h^*)l''(h^*)}{(r + al'(h^*))^2} + \frac{ag_{hh}(S, h^*)\theta}{r + al'(h^*)} - \frac{ag_h(S, h^*)\theta(r + al''(h^*))}{(r + al'(h^*))^2} < 0$$

The derivative of  $B$  with respect to  $\theta$  is equal to

$$B_\theta = \frac{ag_h(S, h^*)}{r + al'(h^*)} > 0$$

Like the previous case, the comparative static of the steady state head level with respect to

limu value is positive, i.e.,  $\frac{\partial h^*}{\partial \theta} = -\frac{B_\theta}{B_{h^*}} > 0$ . In either case (either desalination is used or not),

the steady state head level is increasing with the value of limu.

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